Generalized Samarskii–Ionkin problem for partial differential equations

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ABSTRACT

The generalized Samarskii–Ionkin problem is a non-self-adjoint non-local problem with respect to the selected (spatial) variable, which belongs to the class of general non-local problems in the sense of Steklov–Samarskii [1], [2]. In 1977, N.I. Ionkin [3] proposed an original method for studying the solvability of a model non-local problem for the heat conductivity equation, based on representing the solution as a functional series with respect to a biorthogonal system of functions. This method was subsequently used many times to study the solvability of non-local problems with the conditions [3] for various classes of differential equations. But, on the other hand, the method [3] could not always be applied effectively.

New methods for studying the solvability of non-local problems with the Samarskii–Ionkin conditions - the splitting method and the method of a priori estimates - were proposed in 2023–2024 in the works [4], [5]. This report is devoted to these methods and recent results on the solvability of non-local problems with the generalized Samarskii-Ionkin condition.

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Spectral Volterra equation

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ABSTRACT

Let V be the Volterra operator on $L^2(0,1)$, $(Vf)(x) = \int_0^1 f(s)ds$ and M be the multiplication operator $(Mf)(x) = xf(x)$, where $f \in L^2(0,1)$, $0 \le x \le 1$. It is connected with V by the well-known 'Volterra equation' $[M, V] = V^2$, where $[M, V] = MV - VM$ is the commutator of elements M and V. It is easy to see that elements x, y of an arbitrary Banach algebra A, satisfy $[x, y] = y^2$, then $[x, y^n] = ny^{n+1}$ holds for $n \in N$.
We study a characterizations of the classical Volterra operator based on a simple relation between its singular numbers and the eigenvalues of its imaginary part.

Keywords: *Volterra operator, spectrum.*

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Issues of development of artificial intelligence systems in problems of group pursuit and protection

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ABSTRACT

This article discusses models of group behavior. Participants in antagonistic games are divided into three roles: pursuers, targets, and defenders. Scenarios for each of these games can be very diverse. The basic scenario of the article is the motionless location of targets. The pursuers strive to hit the targets along specified, previously determined trajectories. The group of defenders strives to hit all targets using various optimization criteria. Participants will each see their own picture of the events taking place depending on their roles. This article discusses the creation of libraries of algorithms for each role based on their own set of rules. The possibilities of formalizing the rules for digital representation are considered. The possibilities of formalizing the evasion process are also explored. The mathematical apparatus and the library of developed algorithms can be in demand by software developers for autonomous robotic systems.

Keywords: *model, pursuer, target, defender, trajectory, scenario, optimization criteria, rules.*

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Meagre-expansive flow

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ABSTRACT

In this paper, we generalize the notion of expansivity of flow on compact metric space to meagre-expansivity. Also, we talk about some properties and prove that conjugacy invariant of meagre-expansiveness.

Modeling of gas detonation in explosion chambers

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ABSTRACT

When solving problems related to the strength of explosion chamber walls, it is crucial to model the pressure fields that arise due to the detonation of gas mixtures inside the chamber. Two cases are considered: one assuming instantaneous detonation of the mixture and another assuming a finite propagation speed of detonation waves. It is shown that taking into account the finite propagation speed of detonation waves is necessary, as in this case, the resulting pressure can exceed the pressure calculated under the assumption of instantaneous detonation by more than 7.5 times.

Keywords: *Detonation of a gas mixture, ideal gas equation of state, first law of thermodynamics, gas heat capacity, finite propagation speed of detonation waves, detonation wave front, approximate space-time diagram, pressure fields*

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Extensions of some iterative methods to the multidimensional case

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ABSTRACT

In this paper, we propose extensions of several p-order methods (p = 4, 6, 7) to the multidimensional case. It is shown that these are genuine extensions with good convergence properties. Comparisons are made with other methods based on the efficiency index and CPU time. Numerical experiments are included to confirm the theoretical results and the efficiency of the methods.

Keywords: *unstructured mesh, WENO*

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The mixed boundary-value problem for the nonlinear hyperbolic-parabolic equation of a thermal conductivity for the bounded plates

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ABSTRACT

The article discusses a mathematical model and a finite-difference scheme for the heating process of a plate limited in two spatial variables. The disadvantages of using the classical parabolic heat equation for this case and the rationale for using a mixed equation are given. The implicit difference scheme uses an integro-interpolation method to reduce errors. The quasilinear scheme is used to solve an equation with the nonlinear thermal conductivity coefficient. The first boundary conditions are on the left boundary and on the right boundary of the plate on the space variable x. The third boundary conditions are on the upper and lower boundaries on the space variable y. Initial conditions are specified. The heat source in the parabolic part of the equation is 0, and in the hyperbolic part of the equation, sharp heating begins. The problem is solved numerically in the Mathcad-15 package using the locally onedimensional scheme. The paper presents the results of the calculation program for the finite plate in the form of graphs and tables of the temperature field, and a certificate of state registration of the program is received.

Keywords: *Hyperbolic heat equation, Nonlinear mixed equations, Finite difference method, Mixed boundary condition, Heat balance*

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System dynamics approach for reducing air pollution in Ulaanbaatar city

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ABSTRACT

It has been observed that air pollution represents a significant concern in Ulaanbaatar, the capital city of Mongolia. The harsh climate of the region contributes to elevated levels of pollution, particularly during the cold winter months when the consumption of unprocessed coal by residents in homes and ger areas is notably high. Historically, air pollution was not a critical issue in sparsely populated areas; however, recent trends indicate that it has escalated to dangerous levels. This increase is attributed to the migration of rural residents to Ulaanbaatar, resulting in a higher concentration of the population within the city. The primary objective of the ongoing study is to investigate the current state of air pollution in Ulaanbaatar. This includes identifying the sources of pollution and analyzing the changes and trends in morbidity and mortality rates that are believed to be influenced by air quality. In conclusion, the findings of this study are expected to provide valuable insights into the dynamics of air pollution and its impact on public health, thereby informing future policy decisions aimed at mitigating this pressing issue.

Keywords: *System dynamics model, experimental simulation, air pollution, health*

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Method of improving controls in nonlinear by a control systems with constraints based on the fixed point problem

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ABSTRACT

For a class of nonlinear by a control optimal control problems with constraints, the problem of nonlocal improvement of admissible control with preservation of all constraints is considered. The conditions for improving admissible control are presented as a system of functional equations in the control space with an additional algebraic equation, which is interpreted as a fixed point problem of a special control operator. To solve the resulting fixed point problem, an appropriate iterative process is constructed.

Keywords: *nonlinear by a control system, nonlocal control improvement, fixed point problem, iterative process*

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Numerical study of the buckling of a polymer composite material

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ABSTRACT

The article addresses the issue of numerical analysis of the buckling behavior of a polymer composite material. The analysis was performed using the Ansys software package based on the finite element method. Maximum stresses in each layer were obtained, taking into account large nonlinear displacements and failure criteria of the composite material.

Keywords: *Polymer composite materials, buckling, Ansys, strength*

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L^2-decay estimate of the solutions to Schrödinger equations with potential and dissipative nonlinearity

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ABSTRACT

We consider the initial value problem of nonlinear Schrödinger equation:

$$\begin{cases} i\,\partial_t u = -\frac{1}{2}\Delta u + V(x)u + \lambda\,|u|^{p-1}u \\ u(0\,,x) = u_0(x) \end{cases}, \qquad \text{(NLS)}$$

where $t \in \mathbf{R}_+$, $x \in \mathbf{R^n}$ $(n \geq 1)$, i is the imaginary unit and $u = u(t,x)$ is a complex valued unknown function. The real-valued potential V belongs to $L^\infty(\mathbf{R^n})$. In the nonlinearity, λ is a complex constant denoted by $\lambda = \lambda_1 + i\lambda_2$ with $\lambda_1 \in \mathbf{R}$, $\lambda_2 < 0$. The power of nonlinearity is assumed to be critical or sub-critical in the sense of scattering, i.e., $1 < p \leq 1 + 2/n$.

Since the model described by (NLS) contains the dissipation caused by $\lambda_2 < 0$, we are interested in the decay estimate of $\|u(t,\,\cdot\,)\|_{L^2}$, which is related with the energy of the electric field. As previous mathematical results, Hayashi-Li-Naumkin [2] proved the L^2 -decay estimate of the solution under the strong dissipative condition (SDC), i.e., $\lambda_2 < 0$ and $(p-1)|\lambda_1| \leq 2\sqrt{p}|\lambda_2|$. Gerelmaa-Kita-Sato [1] succeeded in the relaxation of (SDC), and obtained the L^2-decay estimate under $\lambda_2 < 0$ and $\lambda_1 \geq 0$.

In this talk, we will present the L^2-decay estimate of the solution under the presence of potential V.

Keywords: *Dissipative nonlinear Schrödinger equation, Decay estimate.*

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The stability of state model of lane-change motion for the autonomous vehicle

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ABSTRACT

The production and technological advancement of vehicles with automated driving systems has accelerated rapidly. Over the past decade, many vehicles have incorporated automation features designed to reduce driver workload and enhance safety. The widespread use of fully autonomous vehicles is quickly approaching reality. Researchers are continually enhancing technologies that enable autonomous vehicles to change lanes, adjust speed, detect obstacles, and make optimal short-term driving decisions based on specific road conditions. Efforts are also focused on mathematically optimizing vehicle behavior under various test scenarios. For instance, maintaining a constant speed in autonomous vehicles can minimize frequent braking and acceleration, which is crucial for improving passenger safety and reducing fuel consumption. Here, we consider the lane-change movement of a self-driving car with respect constant speed. The lane-change motion process is a combination of position shifting and turning by an angle. In this study, the linear and angular velocities of vehicle rotation were determined using a continuous-time state equation system, and their relation was determined using state model. And we analyzed the stability of the state model.

Keywords: *Self-driving car, Slip angle, Steering, Cornering force, State equation*

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Calculation of aerodynamic characteristics of an arbitrary profile at non-stationary flow

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ABSTRACT

In this work, the aerodynamic characteristics of arbitrary profile, oscillating as a solid body with a small amplitude according to some harmonic law in non-stationary attached flow by flat potential flow of an incompressible fluid of, is calculated. The method of replacement of the Fredholm integral equations of the second kind relative to the value of the stationary velocity and the amplitude value of the non-stationary part of the relative velocity with system of the algebraic equations based on replacement of a profile with the entered N-square with a length of the party of an order of size and the internal corner close to π method developed earlier by the authors is used. The critical current line of a stationary flow required for calculating of improper integrals for non-stationary part of the relative velocity is defined as a solution to the Cauchy problem for the value of the stationary velocity. The non-stationary aerodynamic characteristics of torsional and bending vibrations of the «TsAGI» family profiles at different relative thickness, Strouhal numbers and twist angle were calculated.

Keywords: *Arbitrary profile, aerodynamic characteristics, non-stationary flow, ideal incompressible fluid, Fredholm integral equation of the second kind*

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Mathematical analysis of electrocardiograms for control signal development

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ABSTRACT

To develop a control signal for an electrocardiograph (ECG), it is essential to define its properties, form, size, and numerical parameters. Since the control signal must match the rate and characteristics of the ECG device's input signal, we conducted mathematical processing and analysis on electrocardiograms used as input signals.

Our objective was to determine the most suitable control signal by applying Fourier transformation to these signals. Traditionally, research on electrocardiograms has focused primarily on clinical analysis. However, as medical technology and equipment continue to advance, this approach has become insufficient.

To address this gap, we initiated a comprehensive study of electrocardiographic signals using specialized software. In this study, we employed MathCAD to perform Fourier transformation and analyze ECG signals mathematically, ensuring an accurate and reliable basis for control signal development.

Keywords: *Electrocardiogram (ECG), signal processing, ECG equipment calibration, tools and analysis*

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The fourth boundary value problem for a hyperbolic-parabolic equation with a variable heat conductivity coefficient and nonlinear radiation source

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ABSTRACT

The object of the study is a mathematical model of an electric arc that occurs in high-voltage networks during short circuits on the contacts of power switches. Such an arc occurs due to a breakdown of the dielectric layer between the conductors or contact of one of them with a current-carrying element. In this case, an arc fault is a low-temperature plasma that is formed now of current passage through the air gap between the conductors, which leads to the release of a large amount of heat and light radiation. For this high-intensity thermal process on the gap of disconnected contacts of a power switch and subsequent emergency extinguishing of the arc, a mathematical model is derived in the context of thermodynamics, which is the fourth boundary value problem for the mixed equation of heat conductivity. In the model under consideration, light radiation of heat is represented as the Stefan-Boltzmann law, i.e. the product of the thermal radiation coefficient and the temperature to the fourth power as a source of internal heat. In these short-term transient processes, classical parabolic models of heat conduction based on the usual Fourier theory create gross distortions of temperature fields. Hence, for high-intensity switching processes, the classical theory of transfer becomes unfair and it is necessary to use locally nonequilibrium methods of describing such systems, which rely on the generalized Fourier law and lead to hyperbolic-parabolic equations. The problem posed is solved numerically by the integro-interpolation method in the MathCad-15 package.

Keywords: *hyperbolic heat equation, nonlinear mixed type equations, finite difference method, third boundary condition, heat balance*

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An unsupervised anomaly detection in multivariate time series: Detecting sudden stratospheric warming events

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ABSTRACT

Sudden stratospheric warming (SSW) events represent intense meteorological phenomena characterized by rapid and significant temperature increases within the polar stratosphere. These events can have a considerable impact on extreme weather conditions in mid-latitude countries. However, research on the estimation and detection of SSW remains limited. This study aims to develop a novel method for detecting SSW events using unsupervised deep anomaly detection algorithms. By leveraging unsupervised learning, patterns and structures in data can be rec- ognized, facilitating the development of predictive models without prelabelled datasets. The long short-term memory-autoencoder (LSTM-AE) with correlation weighting function model were used for the predicting and extreme value distributions (EVD) were applied for detecting anomaly. National Centres for Environmental Prediction (NCEP) and the National Centre for Atmospheric Research (NCAR) reanalysis1 temperature and zonal wind data were used for model training, whereas ERA5 reanalysis data were used for validation. This study focuses on the Northern Hemisphere (60°N-90°N) at the 10 hPa pressure level from 1979–2023. The results of this study indicate that the LSTM-AE-peaks over threshold (LSTM-AE-POT) hybrid model with dynamic correlation weighting loss function is the most effective for accurately predicting separate and joint fluctuations to detect the SSW events. Performance evaluation is conducted using confusion matrix-based metrics and temporal consistency metrics. The results demonstrate that the LSTM-AE-POT hybrid model identifies long-term, persistent anomalies with high accuracy and consistency. Additionally, the results of the validation confirm the model's stability and smooth detection performance for SSW events.

***Keywords**: Unsupervised deep learning, Autoencoder, Anomaly detection, Event detection, Multivariate*

Fixed point methods for finding degenerate extremal controls

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ABSTRACT

In the class of control-linear optimal control problems, new forms of conditions of the maximum principle are constructed in the form of fixed-point problems in the control space. The fixed-point problems under consideration allow one to introduce new equivalent definitions for degenerate extremal controls and construct new methods for searching for degenerate extremal controls. The proposed fixed-point methods provide an unambiguous definition and admissibility of the values of successive approximations of degenerate extremal controls and allow one to strictly improve non-optimal degenerate extremal controls, unlike the known gradient methods.

Keywords: *linear control optimal control problem, maximum principle, degenerate control, fixed point problem*

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Decay estimate and asymptotic behavior of the solutions to dissipative nonlinear Schrödinger equations with time-dependent amplification

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ABSTRACT

We analyze the decay estimate and asymptotic behavior of solutions to a dissipative nonlinear Schrödinger equation with time-dependent amplification. We determine the precise range of the nonlinear power α and amplification coefficient gamma γ, ensuring global existence and asymptotic behavior specifically within the range $\frac{1+2\gamma+\sqrt{4\gamma^2-28\gamma+33}}{4(1-\gamma)} < \alpha < \frac{2}{1-\gamma}$ for $\gamma \in (0,1)$. Our findings extend existing results by incorporating the influence of time-dependent amplification on the decay rate and long-time behavior of solutions.

Keywords: *dissipative nonlinear Schrödinger equations, subcritical nonlinearity, decay estimate.*

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On one approach to solving the problem of the extremum of the system final state norm

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ABSTRACT

The problem of finding the extremum (minimum or maximum) of the system final state norm is transformed into a finite-dimensional problem in the class of piecewise constant approximations of control. In the transformed problem, a necessary optimality condition is constructed in the form of a fixed point problem for a special control operator. The proposed approach allows one to apply the well-known theory and methods of fixed points to search for extremal controls. Examples of searching for extremal controls are considered and a comparative analysis of the proposed fixed point approach with known methods is carried out.

Keywords: *Problem of the extremum of the system final state norm, discrete-continuous controlled system, extremal control, fixed point problem*

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Comparison of force of mortality models, application

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ABSTRACT

The force of mortality model, also known as the mortality force or hazard function, represents the instantaneous rate of death for an individual at a given age. It is widely used in actuarial science, demography, and life insurance to model mortality and calculate life expectancy or premiums. Various models can describe the force of mortality. Below is a comparison of some common force of mortality models and their applications.

Multiplicative model of the dynamics of regional macroeconomic indicators

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ABSTRACT

In various modifications of macroeconomic models, dynamic processes are studied at the aggregate, sectoral and inter-sectoral levels.

An aggregated multiplicative model of a regional economic system is considered, which, with a fixed sectoral structure, allows for an effective analysis of the dynamics of the main macroeconomic indicators. The accelerator of gross output and regional income is the accumulation of fixed capital, which, through the indicators of material intensity and capital intensity, determines the technological rate of growth of the final product and regional income. An aggregated dynamic model of regional income reproduction is constructed in the form of a first-order linear differential equation. The growth rate of non-production consumption is set as the control parameter, the change in which in scenario calculations determines the dynamics of all macroeconomic indicators, as well as the values of savings, accumulation and consumption rates. In cases of deviation of the control parameter from its calculated threshold values, we obtain a trajectory of macroeconomic growth with a monotonically changing accumulation rate, increasing or decreasing growth rates of all macroeconomic indicators.

Keywords: *gross output multiplier, material intensity, capital intensity, aggregated dynamic reproduction model, first-order differential equation, technological growth rate, accumulation rate, consumption growth rate, trajectory.*

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The effects of intellectual ability on social development, and its empiric analysis

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ABSTRACT
The primary causes of the modern era's rapid development are human intelligence and its effects. Scholars lately tries to explain economic development by intellectual ability, mostly focusing on IQ levels of a nation. The goal of the research is to define the effects of human development, especially, their intellectual ability on the socio-economy. Based on data from sampled countries, we first evaluated the relationship between levels of intellectual capacity and inequality before using a chi-squared test to conclude. We then extended the Ramsey model in our study and performed econometric estimates between independent variables that reflect intellectual capacity and the Human Development Index (HDI). We looked at each country's development classification and found that there was a positive correlation between high-income and upper-middle-income countries, but a negative correlation between lower-middle- and low-income countries. The human development index or social development index has the most effect on people's IQ levels, intellectual ability, or IQ levels. Other indicators, like the knowledge estimation evaluation index or the innovation system, research sector, and ability to distribute knowledge, are better for the economy.

Keywords: *Intellectual ability, model of Ramsey, economics of knowledge, social economic development*

Using automated valuation models for real estate valuation

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ABSTRACT

Asset valuation is not only a fundamental tool in economic relations but also an integral part of asset utilization, management, and planning for all participants in these relationships. Until now, traditional approaches relying on 100% involvement of appraisers have remained dominant, but with technological advancement, there is an increasing trend toward automation. Automated valuation models offer professional appraisers advantages in reducing costs, improving predictive capabilities, and increasing computational precision, while mathematical calculations and statistical modeling enable the forecasting of real estate market prices based on historical transactions. This research examines the theoretical concepts, historical development, current state, and trends in real estate valuation and automated valuation models. The research has developed a four-stage process mapping that defines the boundaries between appraiser activities and automated operations in developing value predictions for real estate valuation (hereinafter REV). It also determines how to utilize values predicted by automated valuation models (hereinafter AVM). Using the AVM calculations, the appraiser makes decisions on selecting the actual value by applying coefficients that consider the property's specific characteristics and materiality. This research work is valuable for banks, insurance companies, financial institutions, valuation companies, and professional appraisers in preparing information for REV, developing pipeline processes for automated valuation models, and applying the proposed model for calculating real estate values, risk assessment, and calculating tax bases.

Keywords: *data, price index, artificial neural network, information system, artificial intelligence*

Generating function method for constructing new iterations with p (p = 5, 6) order of convergence

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ABSTRACT

In this paper we propose a generating function method for constructing new two and three-point iterations with p (p = 5, 6) order of convergence. This approach allows us to derive a new family of the optimal order iterative methods that include well known methods as special cases. The necessary and sufficient conditions for p-th order convergence of the proposed iterations are given in terms of parameters τ_n and α_n. Several numerical results are given to demonstrate the efficiency and performance of the presented methods and compare them with some other existing method.

Keywords: *Nonlinear equations, higher-order iterative methods, order of convergence*

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Methodologies for determining the impact of surface subsidence at the intersection of cave mining

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ABSTRACT

Cave mining is indeed a bulk underground mining method, specifically designed for extracting large, low-grade ore bodies economically. It involves allowing a portion of the ore body to cave under its own weight, which facilitates mass production at low operational costs. Cave mining involves creating an environment where the ore body is undermined, causing it to collapse under its own weight. This controlled collapse allows for efficient extraction of large volumes of ore from underground. It is a useful technique to extend the life of large deposits previously mined by open pits, and it is a method increasingly proposed for new mines around the world. Mining subsidence refers to the gradual sinking or sudden collapse of the ground surface due to the extraction of underground resources, typically from mining activities. This phenomenon occurs when the removal of minerals, coal, or other materials creates voids below the surface, leading to a reduction in the support of the overlying earth layers. Block caving-induced subsidence can be detected and analyzed using various methods. We have done comparison of the Probability integration method and the Time function model based on the expression of the Knothe model.

Keywords: *cave mining, subsidence, time function model, probability integration method*

Nonstandard finite difference scheme for SIRS epidemic model

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ABSTRACT

Mathematical model of infectious disease spread plays a crucial role in understanding the real epidemiological situation, predicting the dynamics of transmission, and making optimal decisions. In the SIRS model, although infected individuals recover, they lose immunity after a certain period and become susceptible to reinfection, which is a common phenomenon in practice. Therefore, in this study, we constructed a new type of non-standard finite difference scheme for the SIRS model and investigated its properties. The proposed scheme is independent of the computational step size and preserves important qualitative features such as population conservation law and positivity of solutions. Our scheme not only ensures dynamic consistency but also provides a more accurate numerical representation of the transmission process compared to other methods.

Keywords: *SIRS model, Non-standard finite difference scheme, dynamic consistency, Epidemic model*

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Mathematical models and applications in mining

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ABSTRACT

Mathematics plays a role in a variety of mining applications, from mine planning and operational optimization to understanding complex material behaviours and predicting outcomes under uncertain conditions. Mathematical models are essential tools for analysing geological data, optimizing resource extraction processes, and ensuring the safety, efficiency, and environmental sustainability of mining operations. As the mining industry faces increasing demands for sustainable practices and risk mitigation, future applications are increasingly focused on the integration of fluid flow theory—particularly groundwater modelling and hydrogeological analysis. These approaches support critical areas such as groundwater control, mine dewatering, tailings management, and environmental impact assessment. By employing mathematical frameworks including differential equations, numerical simulation, and data-driven modelling, we can better understand subsurface fluid dynamics and their interactions with geological formations. This presentation explores how advanced mathematical modelling of fluid flow can inform next-generation mining strategies, improve decision-making, and contribute to safer and more responsible resource development.

Keywords: *Mathematical modeling, mining optimization, fluid flow theory, groundwater modeling, hydrogeology*