

On gap functions for equilibrium problems

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ABSTRACT

The equilibrium problem defined by the Nikaido-Isoda-Fan inequality encompasses various models including optimization problems, variational inequalities, fixed point problems, Nash equilibria, and others as special cases. This presentation provides a thorough overview of recent advancements in the theory of duality and gap functions for equilibrium problems ([1],[3-5]) and quasi-equilibrium problems [2].

Keywords: *Equilibrium problems, Gap functions, Conjugate duality, Convex optimization, Variational inequality*

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A survey on the physics of two dimensional (2D) materials

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ABSTRACT

In the late 2004, more than two decades ago, graphene (shown in the above figure with a pictorial representation), a truly two-dimensional system has been discovered and identified. Just few years later the discoverers of graphene were awarded the Nobel prize in Physics. Since then, research on graphene and materials akin has boomed, both from the fundamental and application point of view. Besides graphene now there is a large family of two-dimensional materials that include, to mention the most relevant, Transition metal dichalcogenides (TMD), Transition metal trihalides, M-Xenes etc. The unique properties of such materials can be further combined via the fabrication of two-dimensional hetero structures obtained by Controlled stacking of monolayers of different 2D materials. This can lead to exotic properties of the so-formed hetero structure (like superconductivity in twisted graphene). In this lecture, I will survey, few examples of the Research carried out in the last fifteen years in my group on graphene and related materials. Result will be presented on the mechanical exfoliation of graphene and TMD [1]. The band structure of graphene and its experimental evidence (using Angle Resolved Photoelectron Emission Spectroscopy) of features due to electron phonon coupling will be presented to give an example of fundamental studies [2]. For TMD the focus will be on Molybdenum disulphide, and its optical properties investigated with Raman Spectroscopy and Photoluminescence spectroscopy [3]. Then the physics of Transition metal trihalides will be considered with focus on CrCl_3 and its related extrinsic phases [4]. An outlook then will be presented on the physics of 2D heterostructures.

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Recent advances and future perspectives in the recovery of rare earth elements from natural resources in Mongolia

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ABSTRACT

The significance of rare earth elements to the global economy and their utilization in high-tech technology and industries, including energy, power, transportation, medicine, electronics, chemicals, and petroleum, are strongly required to secure sustainable and responsible REE supply and resource management.

Mongolia has been attracting interest from foreign investors in partnering on REE exploration and development due to its rich and potential REE resources, which are Mushgia Khudag, Khotgor, Lugiin Gol, Khalzan buregtei, Ulaantolgoi, Shartolgoi, and Ulaan Del. It indicates that Mongolia has emerged as a key and noteworthy player in the global rare earth market because of its alternative REE resources.

Recent advances in recovering rare earth elements from their ore deposits in Mongolia, namely Mushgia Khudag, Khotgor, Lugiin Gol, and Khalzan buregtei have shown promising results in improving efficiency, sustainable production, and REE extraction rates. The results of REE recovery from REEs-rich apatite in Mongolia are summarized in this paper.

Further research and development in innovative REE extraction and recovery processes will be significant challenges toward maximizing the potential of Mongolia's REE ore deposits by ensuring a sustainable supply of REEs and minimizing the environmental impact of REE mining.

Keywords: Rare earth elements, ore deposit, Mushgia Khudag, REE recovery, REE mining.

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Geometric lower bounds for Laplacian eigenvalues

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ABSTRACT

In this talk, we present new results concerning lower bounds on the first eigenvalue for the Laplace operators, especially under variable-component Robin boundary conditions. Our analysis focuses on convex domains, where we not only derive novel results for the Robin Laplacian, but also improve existing bounds over those for the Dirichlet Laplacian. The methodology adopted in our study is further extended to establish new lower bounds on the first eigenvalue of the polyharmonic Dirichlet operator. In addition, if time permits, we discuss a simple and elegant proof of the Poincaré inequality specific to Robin Laplacians, based on an equality framework.

The goal of this talk is to present these advances, providing a deeper understanding and expanding the existing knowledge base in the field of eigenvalue lower bound problems. The key ingredients in our proofs are taken from [1]-[3].


This talk is based on our joint work with Rupert L. Frank (Caltech and LMU Munchen) and Ari Laptev (Imperial College London).

Keywords: *Robin Laplacian, Polyharmonic operator, Lowest eigenvalue, Hardy inequality, Poincaré inequality.*

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The AI technologies in education: Pros and Cons

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ABSTRACT

Artificial Intelligence (AI) technology powered all fields of science and all levels of education. Nowadays AI is the hotspot in discussion among scholars, professors and researchers. UNESCO recommends focusing on a human-centered approach to AI [1]. The AI4Science consortium is established by the University of Amsterdam and develops and uses machine learning techniques in fields like: molecular biology, chemistry and astrophysics[2]. AI4Science at Caltech and University of Chicago Established in 1986 and now it extends in various directions: AI4Helath, AI4Earth, AI4Exploration, AI4Sumlation, Ai4Speed[3]. AI4Science research group at Free University of Berlin focuses on interdisciplinary research activities [4]. The Microsoft has AI4Science research group which mostly works in project-oriented ways [5]. AI can be useful in problems: assistance, speed, individualization, context and administration tasks. But privacy, security, reduced human interaction, costs and unpredictability are can be side of the AI technologies in education.

Keywords: *Artificial Intelligence, advantages of AI, side effect of AI, Higher education, AI and Science.*

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Directional thermal conversion of lignite to chemical and carbon-based materials

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ABSTRACT


The classified utilization of low rank coal and coal to chemicals is one of the major strategic needs in China. The thermochemical conversion of lignite and other low-grade resources were comprehensively investigated, and the mechanism of the organic oxygen transformations during thermochemical conversion was proposed. It was found that the selective conversion of lignite to valuable light aromatics can be achieved via in-situ catalytic fast pyrolysis. The aromatization mechanism of macromolecular and small molecular compounds in lignite pyrolysis volatiles, effectively the coupling mechanism during reforming the pyrolysis volatile was revealed. In addition, a high reactivity of nickel supported on lignite-derived activated carbon catalysts were prepared and a theory and process system for selective reforming of volatiles to specific products were established. This work is aimed to provide theoretical basis for directional thermal transformation of lignite.

Keywords: *low-rank coal, aromatics, activated carbon, pyrolysis.*

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SmFe₁₂-based compounds: Bridging theory and experiment for next-generation magnets

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ABSTRACT

The search for rare-earth-lean permanent magnets has brought SmFe₁₂-based compounds to the forefront due to their exceptional theoretical potential for high magnetization, anisotropy, and Curie temperature. However, realizing these properties in practice remains challenging due to phase stability issues and synthesis limitations. In work, we will present a comprehensive overview of recent advances in SmFe₁₂ research, combining experimental breakthroughs and theoretical insights.

On the experimental side, we demonstrate the successful epitaxial growth of SmFe₁₂ thin films using molecular beam epitaxy (MBE), highlighting the role of flux ratio control, capping layers, and strain engineering in achieving phase-pure, oriented structures. Magnetometry and structural analysis reveal anomalous hysteresis behaviors and tunable magnetic anisotropy, particularly under strain or elemental substitution.

From the theoretical perspective, we employ density functional theory (DFT) to explore the effects of dopants (e.g., Ti, B, and rare-earth substitutions) and strain on the stability, electronic structure, and magnetic properties of SmFe₁₂. Our calculations provide guidance for optimizing coercivity and enhancing intrinsic performance metrics. By integrating experiment and theory, we establish a roadmap for engineering SmFe₁₂-based compounds as viable candidates for high-performance, rare-earth-efficient magnets.

Stochastic optimal control of heating systems with a geothermal energy storage

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ABSTRACT

We consider a residential heating system with renewable and non-renewable heat generation and different consumption units and investigate a stochastic optimal control problem for its cost-optimal management. As a special feature, the heating system is equipped with a geothermal storage that enables the intertemporal transfer of thermal energy by storing surplus heat for later use. In addition to the numerous technical problems, economic issues such as cost-optimal control also play a central role in the design and operation of such systems. The latter leads to challenging mathematical optimization problems, as the response of the storage to charging and discharging decisions depends on the spatial temperature distribution in the storage. We take into account uncertainties regarding randomly fluctuating heat generation from renewable energies and the environmental conditions that determine heat demand and supply. The dynamics of the multidimensional controlled state processes is governed by a partial, a random ordinary and two stochastic differential equations. We first apply a spatial discretization to the partial differential equation and use model reduction techniques to reduce the dimension of the associated system of ordinary differential equations. Finally, a time-discretization leads to a Markov decision process for which we apply a state discretization to determine approximations of the cost-optimal control and the associated value function.