MATHEMATICS & COMPUTATIONAL SCIENCE

New Principles of Nonconvex Optimization

Alexander Strekalovsky[1,*]

[1]*Institute for System Dynamics and Control Theory, Russia*

**Correspondence author. Email: strekal@icc.ru, ORCID: 0000-0002-4664-6961*

ABSTRACT

A new Global Search Theory (GST) was developed for nonconvex optimization problems with equality and inequality constraints and a cost function given by DC functions. The Foundation of the GST is Global Optimality Conditions (GOCs) of new type. In addition, the new GOCs generalize the results of the Classical Optimization Theory. It is worth noting that any optimization problem with continuous data on a compact set can be approximated at any desirable accuracy by a corresponding DC optimization problem. New results provided foundation for development of absolutely new original tools for a Global Search in a general DC optimization problem, and, in particular, for the canonical DC optimization problems, such as convex maximization, reverse-convex and DC-minimization problems.

We also study the convergence properties of the LSMs and the GSS. The elaboration of the Global Search Theory (GST) allows us find the solution of the following applied problems, some of which, according to the opinion of distinguished experts in optimization, represent the challenges of the 21st century.

Keywords: *Global Search Theory (GST), Global Optimality Conditions (GOCs), Global Search Scheme (GSS), Special Local Methods (LSM).*

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Decay estimate and asymptotic behavior of solutions to the system of Schrödinger equations with cubic dissipative nonlinearity

Naoyasu Kita[1,*]

[1]*Kumamoto University, Kumamoto, Japan*

**Correspondence author. Email: nlkita@kumamoto-u.ac.jp*

ABSTRACT

This is a joint work with Y. Sagawa and Y. Nakamura. We consider the initial value problem of nonlinear Schrödinger equation:

$$(\mathrm{NLS})\begin{cases} i\,\partial_t \mathbf{u} + \dfrac{1}{2}\partial_x^2 \mathbf{u} = \mathbf{f}(\mathbf{u}) \\ \mathbf{u}(0) = \mathbf{u}_0 \end{cases},$$

where $t \in \mathbf{R}_+$, $x \in \mathbf{R}$, i is the imaginary unit and $\mathbf{u} = \mathbf{u}(t, x) = (u_1\ u_2\ \cdots\ u_n)^t$is a $\mathbf{C}^n$-vector valued unknown function. The nonlinearity $\mathbf{f}(\mathbf{u})$ is a map from the set of $\mathbf{C}^n$-vectors to itself, described as

$$\mathbf{f}(\mathbf{u}) = (f_1(\mathbf{u})\ f_2(\mathbf{u})\ \cdots\ f_n(\mathbf{u}))^t.$$

We assume that the nonlinearity satisfies

(A.1) [gauge-invariance] $\mathbf{f}\left(e^{i\theta}\mathbf{u}\right) = e^{i\theta}\,\mathbf{f}(\mathbf{u})$ for any $\theta \in \mathbf{R}$,

(A.2) [cubic nonlinearity] $\mathbf{f}(\lambda\mathbf{u}) = \lambda^3\,\mathbf{f}(\mathbf{u})$ for any $\lambda > 0$,

(A.3) [dissipative structure] let $\mathbf{A}(\mathbf{u}) = (a_{jk}(\mathbf{u}))$ be a matrix, where the entries are

$$a_{jj}(\mathbf{u}) = \mathrm{Im}\,\frac{\partial f_j(\mathbf{u})}{\partial u_j} + \left|\frac{\partial f_j(\mathbf{u})}{\partial \overline{u_j}}\right| \quad (1 \le j \le n),$$

$$a_{jk}(\mathbf{u}) = \frac{1}{2}\left\{\left|\frac{\partial f_j(\mathbf{u})}{\partial u_k}\right| + \left|\frac{\partial f_k(\mathbf{u})}{\partial u_j}\right| + \left|\frac{\partial f_j(\mathbf{u})}{\partial \overline{u_k}}\right| + \left|\frac{\partial f_k(\mathbf{u})}{\partial \overline{u_j}}\right|\right\} \quad (j \ne k).$$

Then there exists some $\rho > 0$ such that all μ's (the eigen-values of $\mathbf{A}(\mathbf{u})$) satisfy

$$\mu \le -\rho|\mathbf{u}|^2.$$

The equation (NLS) is a generalization of a model in the nonlinear optics. For (NLS), we will derive decay estimate of $\|\mathbf{u}(t,\ \cdot\,)\|_{L^\infty}$. Furthermore the asymptotic behavior of $\mathbf{u}(t,\ x)$ will be considered. In these results, the nonlinear effect explicitly appears.

Keywords: *Schrödinger equations, cubic dissipative nonlinearity, asymptotic behavior.*

Some properties of the Volterra pencils

Tsedenbayar Dashdondog[1,*], Jargalsaikhan Tsogkhuu[1], Buyantogtokh Danzan[1]

[1]*Department of Mathematics, School of Applied Sciences, University of Science and Technology, Ulaanbaatar, Mongolia*

**Correspondence author. Email: tsdnbr@must.edu.mn , ORCID: 0000-0002-4053-7983*

ABSTRACT

We introduce the numerical ranges techniques for some polynomials generated by the Volterra operator. In particular, we present the operator norm, the numerical range, the numerical radius and the accretive properties of the real and the imaginary parts for such polynomials. It seems interesting to point out that a similar procedure applies to the Volterra polynomials.

Keywords: *Volterra, numerical range, numerical radius.*

Using mathematical methods for decision

Enkhbayar Jamsranjav[1,*], Tserendulam Shurenchuluun[1]

[1]*Department of Applied Mathematics, School of Engineering and Applied Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: enkhbayar.j@seas.num.edu.mn, ORCID: 0000-0003-4576-3864*

ABSTRACT

When implementing a system in which students can freely choose to study, it is very important for a New Normal student to correctly calculate the time or workload to spend on learning. One way to improve the management of higher education is to manage the learning process based on optimization of calculation time of a student on course-related activities. Curricula with too much content and high emphasis on each content lead to high student workload. When the workload is too much, the student to get the grade performs the tasks with a lot of memorization without understanding the meaning, copying without thinking, so they learn passively. The property-determined workload is one of the important factors affecting student learning, so it is necessary to optimize calculation student workload.

Keywords: *Student workload, Sphere packing approach, Computation optimization, Credit system, Higher education management, Business administration program*

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Study on global dynamics of a special symmetrically laid composite laminated rectangular plate

Wensai Ma[1,*], Dongxiao Li[1], Fanghao Liu[1], Shufeng Lv[1]

[1]*Department of Mechanics, Inner Mongolia University of Technology, Hohhot, China*

**Correspondence author. Email: wensaima@163.com*

ABSTRACT

The global bifurcation and chaotic dynamics of simply supported orthotropic symmetrically laminated composite rectangular plate is firstly studied by using the generalized Melnikov method. The five-dimensional nonlinear dynamic system of composite laminated rectangular plate is obtained by coordinate transformation theory. The k-pulse Melnikov function of the system is calculated by generalized Melnikov theory, and the chaotic threshold interval of composite laminated rectangular plate system is obtained. Using MATLAB software, the theoretical results are numerically simulated, and the bifurcation diagram, phase diagram and time history diagram of composite laminated rectangular plate are obtained to verify the correctness of the theoretical analysis. Finally, theoretical analysis and numerical simulation further show that there is multi-pulse chaotic motion in the composite laminated rectangular plate.

Keywords: *Composite laminated rectangular plate, bifurcation, chaos, Melnikov method.*

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Generalized integral representation method as applied to numerical simulation of Boussinesq wave

Gantulga Tsedendorj[1], Baljinnyam Tsangia[2], Khongorzul Dorjgotov[3,*]

[1]*Department of Mathematics, School of Arts and Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*
[2]*Department of Mathematics, School of Applied Sciences, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[3]*Department of Applied Mathematics, School of Engineering and Applied Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: khongorzul@seas.num.edu.mn, ORCID: 0000-0002-3331-2516*

ABSTRACT

In this study, we present discretization schemes based on Generalized Integral Representation Method (GIRM) for numerical simulation of the Boussinesq wave. The schemes numerically evaluate the coupled Boussinesq equation for different solitary wave phenomena, namely, propagation of a single soliton, head-on collision of two solitons and reflection of a soliton at a fixed wall boundary. In these soliton interactions, we utilize different Generalized Fundamental Solutions (GFS) along with piecewise constant approximations for the unknown functions. For the case of soliton reflection at a wall, time evolution in GIRM is coupled with the Green's function in order to cope with the complicated boundary conditions that arise from the GIRM derivation. We conduct numerical experiments and obtain satisfactory approximate result for each case of the soliton interactions.

Keywords: *Numerical Simulation of Boussinesq Wave, Soliton Interactions, Generalized Integral Representation Method (GIRM), Numerical Schemes based on GIRM.*

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Stochastic optimal control problem of consumption and pension insurance purchase with uncertain lifetime

Gereltuya Terbish[1,*], Tserenbat Oirov[2], Batsukh Tserendorj[3], Nyamsuren Dorj[1]

[1]*Department of Mathematics, School of Applied Sciences, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[2]*Department of Applied Mathematics, National University of Mongolia, Ulaanbaatar, Mongolia*
[3]*Department of Economics, University of Finance and Economics of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: gereltuya.t@must.edu.mn, ORCID: 0000-0002-6479-6146*

ABSTRACT

We consider a continuous-time model of optimal consumption and pension insurance for a consumer with an uncertain lifetime. In the model, the consumer earns a stochastic wage income during her working life and optimally allocates her income between personal consumption, pension insurance, and securities with a deterministic dynamic return. The consumer's utility and bequest functions are CARA (constant absolute risk aversion). By characterizing the optimality condition of the consumer's problem using the Hamilton-Jacob-Bellman equation, we find the optimal consumption and pension insurance as a function of wealth in closed form. We consider an application of the model while estimating its key elements using real-life data on age-specific population size, labour income, and interest rates. We show that as the absolute risk aversion for consumption increases, consumption and wealth move in the opposite direction. We also present a novel finding that wealth and consumption can be negatively related across consumers with different levels of consumption risk aversion.

Keywords: *Stochastic optimal control, optimal consumption, pension insurance, relative risk aversion, bequest.*

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Impact of representative capacity on the size of parliamentary institution in Mongolia

Tuyagerel Ganbat[1,*], Undrakh Davaadorj[1], Battugs Sukhee[2]

[1]*Department of Political Science, National University of Mongolia, Ulaanbaatar, Mongolia*
[2]*Department of Physics and Mathematics, Mongolian University of Life Sciences, Ulaanbaatar, Mongolia*

**Correspondence author. Email: tuyagerel.g@num.edu.mn*

ABSTRACT
The implementation and consolidation of democracy has been studied from many aspects in Mongolia, and based on modern research methods, it has been studied in detail with many criteria. The diversity of democracy is observed differently in each country, it is necessary to consider actual political observations and evaluations rather than legal regulations in those countries.
This survey was organized on two main levels: public opinion and expert. More than 120 experts working in the field of parliamentary and democracy studies participated in the section for experts , and about 1,200 respondents participated in the section for public opinion. As for the survey question, the research method of the Michigan University Institute
for Social Research and the Center for Political Studies, which was first conducted in 1952 and is used as an internationally popular method, and the method used by Christopher H. Achen in his research work "Measurement representation" were used.
The main purpose of this research report is to determine who should have the right to change the number of members of The State Great Hural /parliament/ of Mongolia, what are the main functions of the member, and the institutional capacity of the parliament through the implementation of representative functions.

Keywords: *Optimal size of parliament, shape of parliament.*

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Synergistic drug combination study with multi-omics data integration

Altankhuyag Avirmed[1], Dolgorsuren Batjargal[2,*], Temuulen Dorjsuren[3], Yansen Su[4], Tseren-Onolt Ishdorj[1]

[1]*Department of Computer Science, School of Information and Communication Technology, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[2]*Department of Information Technology, School of Information and Communication Technology, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[3]*Mongolian National University of Medical Sciences, Ulaanbaatar, Mongolia*
[4]*School of Artificial Intelligence, Anhui University, Hefei, Anhui, China*

**Correspondence author. Email: b.dolgorsuren@must.edu.mn, ORCID: 0000-0002-8795-2762*

ABSTRACT
To understand complex biological processes comprehensively for drug repurposing, it is imperative to use integrative techniques that combine multiple omics datasets to identify the interrelationship between genes, compounds, diseases, biological processes, side effects, and symptoms. Several data crawling tools and methods can be developed for data integration and interpretation with graph techniques and the availability of multi-omics data. In this paper, we propose an approach for integrating multi-omics data for adopting machine learning algorithms and summarize their ability to address applications such as echinococcosis disease-related drug re-purposing in a heterogeneous graph and deriving insights into the data. In addition, we discuss the machine learning task, which is identifying the two-and multi-drug synergy effect used in the analysis of integrated data, and show a developed web-based application for visualizing the result of the proposed algorithm. We evaluate the performance of four drug synergy models, including ZIP, BLISS, LOEWE, and HSA, using synthetic and real drug databases.

Keywords: *Multi-omics data, data integration, drug repurposing, heterogeneous network, drug synergy.*

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On problems related to the higher-order Schrödinger operators

Telmen Enkhtuvshin[1,*], Galtbayar Artbazar[†1]

[1]*Department of Applied Mathematics, National University of Mongolia, Ulaanbaatar, Mongolia.*

**Correspondence author. Email: telmen87@gmail.com, ORCID: 0009-0005-5034-7111*
† Correspondence author. Email: galtbayar@seas.num.edu.mn

ABSTRACT
The scattering theory of higher-order Schrödinger operator H, with sufficiently small potentials, has been extensively studied in specific dimensions. One interesting problem related to the wave operators of higher-order Schrödinger operators is the $L^p - L^p$ boundedness problem. This property allows us to derive properties of $f(H)P_{ac}$ for any Borel function f on the real line, leveraging the interwining properties of wave operators.
Another fascinating problem is the time decay estimation of higher-order Schrödinger operators, which provides $L^p - L^q$ estimates with $\frac{1}{p} + \frac{1}{q} = 1$ for the one-parameter unitary group associated with the operator H. Under certain conditions on the potentials, both of these problems can be reduced to estimating the spectral integral on the half line. A common strategy to tackle this problem is to decompose the integral into the sum of two integrals, known as the low-energy and high-energy parts.
In this study, our focus is on the wave operators of two-dimensional second-order Schrödinger operators with sufficiently small potentials. With some spectral properties for this operator obtained by A. Soffer and others in study of time decay estimation, we consider some problems related to the wave operators.

Keywords: *Scattering theory, higher-order Schrödinger operators, spectral theory.*

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The long run relationship between budget and trade deficit of Mongolia

Naranchimeg Chimedravdan[1,*], Davaasuren Chuluunbaatar[1]

[1]*Department of Fundamental sciences, University of finance and economics, Ulaanbaatar, Mongolia*

**Correspondence author. Email: naranchimeg.ch@ufe.edu.mn*

ABSTRACT

Mongolia has been implemented a budget expansion policy since 2006, but the budget deficit and government debt have been continue to grow. In 2022, the budget deficit reached 0.98 trillion MNT, accounting for around 1.9% of GDP at annual prices, this shows a decrease of 4.8 percentage from the previous year. But the export reached 12,540.0 million in US dollars, an increase of 35.7% equals to 3,299.0 million in US dollars, and import reached 8,704.0 million US dollars, an increase of 27.2% equals to 1,859.0 million in US dollars which means trade balance profit increased by 60.1% or 1,440.0 millions US dollars from last year. Therefore, we use time series data from 2000 to 2022 to determine the long-run relationship between the budget deficit and trade deficit, and within the framework of Mandel-Fleming theory and methodology which is suitable for small open economies such as Mongolia. In this paper an econometric analysis is performed by using Autoregressive Distributed lag model approach and the empirical results we indicate that the budget deficit leads to trade deficit in the long run.

Keywords: *Export, currant account, fiscal deficit, import, integration.*

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Lie symmetry reductions of (2+1)-dimensional generalized KdV equation and multi-component nonlinear Schrödinger equations

Yushan Bai [1,*], Yana Liu[1]

[1]*College of Science, Inner Mongolia University of Technology, Hohhot, China*

**Correspondence author. Email: mbaiyushan@imut.edu.cn, ORCID: 0000-0001-7813-4672*

ABSTRACT

In this talk, we give the Lie symmetries of a (2+1)-dimensional generalized KdV (gKdV) equation and multi-component nonlinear Schrodinger (MNLS) equations in nonlinear mathematical physics. By applying the obtained symmetries, we give the symmetry reductions of the MNLS equations and construct some explicit solutions. Moreover, we also present symmetry reductions of the (2+1)-dimensional gKdV equation and use the Riccati equation expansion method to find some interesting explicit solutions. This talk is based on joint work with Yana Liu.

Keywords: *Lie point symmetries, Symmetry reductions, Explicit solutions, Riccati equation expansion method.*

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DC optimization approach for bimatrix game for finding Berge equilibrium

Mengkezhula Sa[1], Batbileg Sukhee[1], Enkhbat Rentsen[2,*], Battur Gompil[1]

[1]*Department of Applied Mathematics, School of Engineering and Applied Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*
[2]*Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia*

**Correspondence author. Email: enkhbatm@num.edu.mn, ORCID: 0000-0003-0999-1069*

ABSTRACT

The paper deals with finding a Berge equilibrium by DC optimization method for the global solution. We examine Berge equilibrium in the bimatrix game for mixed strategies. Before we proved an existence of Berge equilibrium in the bimatrix game and redused the bimatrix game to a nonconvex optimization problem. In this paper we show that a new formulated optimization problem can be also reduced to D.C. programming so that one can apply the global optimality conditions. Berge equilibrium is a model of cooperation in social dilemmas, including the Prisoner's Dilemma games. The Berge equilibrium concept was introduced by the French mathematician Claude Berge for coalition games and formalized by Zhukovskii in the context of differential games. The main goal this paper is to fulfill this gap and develop theory and algorithm of Berge equilibrium for bimatrix game with mixed strategies. The work is devoted to the existence of Berge equilibrium in a bimatrix game for mixed strategies, its global optimization formulation and its reduction to dc optimization problem and numerical experiments on Berge equilibrium for nonzero sum two person game have been provided.

Keywords: *Berge equilibrium, Nash equilibrium, DC programming, global optimization, global optimality conditions.*

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The search solutions in military events

Sainbayar Davaajav[1], Ankhbayar Gelegbadam[2], Batchimeg Tsendpurev[3,*]

[1]*Department of Natural Sciences, National Defense University, Ulaanbaatar, Mongolia*
[2]*Department of Applied Mathematics, National University of Mongolia,Ulaanbaatar, Mongolia*
[3]*Ulaanbaatar School, National University of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: batchimeg.ts@num.edu.mn, ORCID: 0009-0008-0780-7903*

ABSTRACT
In 1956, the journal of the American Mathematical Society published a short article entitled "Lost at sea" which proposed 3 problems by the famous mathematician Bellman R.
The first problem proposed by Bellman R. was the theory to determine the pattern of the curve with the shortest length among all the search curves. The problem was solved by. Isbell J.R in 1957, and solved again by Gluss B in 1961 by replacing the straight shore mainland with an island with radius S Khaltar D. expanded and solve these problems in various ways in his works by connecting with nomadic, pastoralists and geological prospecting activities.
In this work, we performed a full analysis by considering two models related to the detection of the enemy's front line and firing point, and these models are extensions of the search problems solved by Isbell J.R and Gluss B. To find the optimal solution, we used the concepts of extremal problem theory and plane geometry.

Keywords: *Search solution, extreme theory, mathematical design, and the shortest search curve.*

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Multi–period loan interest rate Nash model with Basel II solvency constraint

Enkhbayar Khangai[1,*], Battulga Gankhuu[2], Batbileg Sukhee[2]

[1]*Department of Mathematics, School of Applied Sciences, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[2]*Department of Applied Mathematics, School of Engineering and Applied Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: eegii33@must.edu.mn, ORCID: 0000-0002-1259-5502*

ABSTRACT

In this paper, we introduce multi–period loan interest rate Nash game models in the banking sector under regulatory solvency constraints. By taking solvency constraint as Basel II and modelling economic condition as AR(1) process, we obtain results regarding the existence of loan interest rate equilibrium. Basel II uses a "three pillars" concept, namely, minimum capital requirements (addressing credit risk, market risk, and operational risk), supervisory review, and market discipline. Process of Basel II attracted a lot of interest in quantitative credit risk models in industry, academia, and among regulators. A sensitivity analysis for the solvency constraint model and some numerical results are presented.

Keywords: *Nash equilibrium model, one factor KMV/Riskmetrics model, Basel II solvency constraint, credit rating, loan interest rate*

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The modified parallel tangent global search algorithm for finding anti-Berge equilibrium in bimatrix game

Enkhbat Rentsen[1,*], Pavel Sorokovikov[2], Tatiana Zarodnyuk[2], Alexander Gornov[2]

[1]*The Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia*
[2]*Matrosov Institute for System Dynamics and Control Theory Siberian Branch of Russian Academy of Sciences Irkutsk, Russia*

**Correspondence author. Email: enkhbatm@num.edu.mn, ORCID: 0000-0003-0999-1069*

ABSTRACT

We examine the problem of finding an anti-Berge equilibrium in bimatrix game based on a global search algorithm. Finding anti-Berge equilibrium equivalently reduces to a quadratic programming with an indefinite matrix and linear constraints which belongs to a class of global optimization. To solve the problem numerically, we develop a modified parallel tangent algorithm. The proposed algorithm uses the one-dimensional nonlocal search procedure based on the Strongin and parabolas methods. Stopping criteria of the algorithm is the sufficient condition of anti-Berge equilibrium. The proposed algorithm is implemented and numerically tested on a collection of bimatrix games.

Keywords: *Game theory, anti-Berge equilibrium, global optimization, parallel tangent algorithm, Strongin method, parabolas method.*

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A novel path integration method for the stochastic response of vibro-impact systems

Liang Wang[1,*]

[1]*School of Mathematics and Statistics, Northwestern Polytechnical University, Xi'an, China*

**Correspondence author. Email: liangwang1129@nwpu.edu.cn*

ABSTRACT

In this talk, we will give a new path integration algorithm specifically for the stochastic vibro-impact system. By introducing the concept of absorption surface and the impact completion condition, the new algorithm can be directly used to study the stochastic response of vibro-impact systems without any non-smooth approximation. The algorithm is suitable for arbitrary recovery coefficients and can retain the non-smooth characteristics of the vibro-impact system. Linear and nonlinear vibro-impact system excited by the Gaussian white noise are used as examples to show the unique results obtained by our method. Finally, the Monte Carlo method is used to verify the efficiency and accuracy of this new algorithm.

Keywords: *Vibro-impact system, Gaussian white noise, Monte Carlo method.*

Invariance properties of time fractional linear diffusion-wave equations

Nomuun Bayanmunkh[1], Khongorzul Dorjgotov[2], Bayarpurev Mongol[2], Gantulga Tsedendorj*[,3], Uuganbayar Zunderiya[3]

[1]*School of Arts and Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*
[2]*Department of Applied Mathematics, National University of Mongolia, Ulaanbaatar, Mongolia*
[3]*Department of Mathematics, National University of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: gantulga@smcs.num.edu.mn, ORCID: 0000-0002-8957-630X*

ABSTRACT

We study a class of time fractional diffusion-wave equations with variable coefficients using Lie symmetry analysis. We obtain not only infinitesimal symmetries but also a complete group classification and a classification of group invariant solutions of this class of equations. Group invariant solutions are given explicitly corresponding to every element in an optimal system of Lie algebras generated by infinitesimal symmetries of equations in the class. We express the solutions in terms of Mittag-Leffler functions, generalized Wright functions, and Fox H-functions. These solutions contain previously known solutions as particular cases.

Keywords: *Time fractional differential equation, diffusion wave equation, invariance properties.*

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Optimal L^2 -decay of solutions to nonlinear Schrödinger equations with a long-range dissipative nonlinearity

Takuya Sato[1,*]

[1] *Faculty of Advanced Science and Technology, Kumamoto University, Kumamoto, Japan*

**Correspondence author. Email: neipia27182818@gmail.com, ORCID: 0000-0002-4660-9669*

ABSTRACT

We consider the optimality of mass decay of solutions to the nonlinear Schrödinger equations with a long-range dissipative nonlinearity in one spatial dimension. We show that the L2 -norms (mass) of any global solutions do not decay more rapidly than (log t)−1/2 , and we also prove that there exists a solution decaying just at the rate of (log t)−1/2 in L2 . This paper is survey based on the paper: Kita-Sato and this work is based on the joint work with Professor Naoyasu Kita (Kumamoto University).

Keywords: *Cauchy problem, Schrödinger equation, Fourier transform, pseudo-conformal transform.*

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On the properties of Bäcklund transformations on Painlevé 6th equations

Amarzaya Amartuvshin[1,*], Davaadorj Balbaan[1]

[1]*Department of Mathematics, National University of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: amarzaya@smcs.num.edu.mn,*

ABSTRACT

In his series of papers Okamoto give detailed explanation of Painlevé equations of types 2-6. On the other hand Kajiwara et.al described Bäcklund transformations for these types of Painlevé equations. In this research we study properties of Bäcklund transformations of 6th Painlevé equations P_6.

Keywords: *Painlevé equations, Bäcklund transformations, Affine Weyl group.*

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Quintic B-spline method for solving Lane-Emden type equations

Zhanlav Tugal[1], Otgondorj Khuder[2,*], Ulziibayar Vandandoo[2], Mijiddorj Renchin-Ochir[3]

[1]*Institute of Mathematics and Digital Technology, Mongolian Academy of Science, Ulaanbaatar, Mongolia*
[2]*Department of Mathematics, School of Applied Sciences, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[3]*Department of Informatics, Mongolian National University of Education, Ulaanbaatar, Mongolia*

**Correspondence author. Email: otgondorj@must.edu.mn, ORCID: 0000-0003-1635-7971*

ABSTRACT
In this work, we propose efficient and high accurate quintic B-spline method for solving Lane-Emden type equations which describes various physical and astrophysics phenomena. The method produces approximate solution in the form of B-spline representation which is very handily from computational point of view. The main advantage of the obtained quintic spline is that it approximate not only the solution of problem but also the derivative of solution with higher accuracy. In fact, we obtain quintic spline approximating the solution to Lane-Emden equations without using the exact solution.

Keywords: *Quintic B-spline, Multi-point finite-difference scheme, Boundary value problem, Solving nonlinear systems, Lane-Emden equation.*

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Goal programming approach to achieve sustainable forest managament: case study in Mongolia

Usukhbayar Enkhjargal[1,*], Battur Gompil[1], Batchuluun Tseveen[2], Gerelbaatar Sukhbaatar[2], Bayanmunkh Norov[3], Atartsetseg Tegshjargal[2]

[1]*Department of Applied Mathematics, National University of Mongolia, Ulaanbaatar, Mongolia*
[2]*Department of Environment and Forest Engineering, School of Engineering and Applied Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*
[3]*Centre for Policy Research and Analysis*

**Correspondence author. Email: eusukhbayar23@gmail.com, ORCID: 0000-0002-0618-1071*

ABSTRACT
Forest resources are the most important natural resources on this earth but they are continuously depleting due to the overgrowth of human population, needs, and industrialization, other development activities. Therefore, the conservation of forest resources has been the main problem for sustainable development and several mathematical methods were used to find the optimal way to achieve it. In this paper, we apply a goal programming method to find the optimal harvest volume for Mongolian forest resources based on data containing volume, growth, and economic evaluations.

Keywords: *Goal programming, sustainable forest management, Mongolian forest.*

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Multi-fidelity reduced-order optimization algorithm

Manyu Xiao[1,*], Piotr Breitkopf[2]

[1]*Xi'an Key Laboratory of Scientific Computation and Applied Statistics, School of Mathematics and Statistics, Northwestern Polytechnical University, Xi'an, China*
[2]*Alliance Sorbonne Universités, Université de Technologie de Compiègne, Laboratoire Roberval, Compiègne, France*

**Correspondence author. Email: manyuxiao@nwpu.edu.cn*

ABSTRACT
The common issue of surrogate models is to make good use of sampling data. In theory, the higher the fidelity of sampling data obtained, the more accurate the approximation model built. However, in practical engineering problems, high-fidelity data may be less available, and such data may also be computationally expensive. On the contrary, we often obtain low-fidelity data under certain simplifications. Although low-fidelity data is less accurate, such data still contains much information about the real system. So, combining both high- and low-fidelity data in the construction of a surrogate model may lead to better representation of the physical phenomena.

In this work, the original principle of Multi-Fidelity Proper Orthogonal Decomposition will consist in projecting the high-dimensional system of equations into a low-dimensional space, with reduced basis vectors. These vectors are updated with incremental "on-the-fly" merges between costly high-fidelity simulation snapshots and more rapid lower-fidelity vectors.
Successful achievements of this work will lead to potential benefits in the fields of large-scale mechanical engineering.

Keywords: *Surrogate models, multi-fidelity, high-dimensional system.*

A new technique for loss reserve estimation

Barsbold Bazarragchaa[1,*], Oyunbold Dudarii[2], Galsan Denzen[1], Dorjkhuu Enkhtaivan[1]

[1]*Department of Applied Mathematics, School of Engineering and Applied Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*
[2]*Ulaanbaatar City Insurance LLC, Ulaanbaatar, Mongolia*

**Correspondence author. Email: barsbold@seas.num.edu.mn*

ABSTRACT
Incurred but not reported (IBNR) losses are estimated using the Chain Ladder Method (CLM). The effectiveness of it depends greatly on the specifics of the insurance product and it is effective when historical compensation data follows certain patterns.
In this research, we give mathematical description to CLM. Based on it we propose criterion, which decides whether CLM is appropriate to apply for given data or not.
Furthermore, we developed a new methodology and tested it for samples were not suitable for the use of classical CLM. Our new approach is a mathematically direct extension to CLM. We had tested the new method using real-world data from domestic insurance company. In case of this data the classical CLM was not suitable for reserve estimation. But the newly developed method estimates IBNR losses more precisely.

Keywords: *Chain ladder method, principal component analysis, linear regression, incurred but not reported losses.*

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A CNN model for SERS spectra detection of SARS-CoV-2 proteins

Ankhbayar Nyamdavaa[1], Kiran Kaladharan[2], Erdene-Ochir Ganbold[3], Anudari Bayarsaikhan[1], Fan-Gang Tseng[2], Tseren-Onolt Ishdorj[4,*]

[1]*Department of Information Technology, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[2]*Department of Engineering and System Science, National Tsing Hua University, Taipei, Taiwan, China*
[3]*Department of Physics, National University of Mongolia, Ulaanbaatar, Mongolia*
[4]*Department of Computer Science, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*

**Correspondence author. Email: tseren-onolt@must.edu.mn, ORCID: 0000-0002-0425-3879*

ABSTRACT
This study proposes a preliminary deep learning model CNN which detects COVID-19 based on the SERS spectra of SARS-CoV-2 proteins generated by AgNPs and SERS based substrate. The proteins we use their Raman spectra are S-protein, VLP protein, Streptavidin proteins in media PBS and untreated saliva and blank signal as well. In order to increase the dataset, we applied data augmentation to the existing spectral data. For the virus detection and protein classifications based on SERS spectra, binary- and multi-class classifications have been carried out by the CNN. The performance of the CNN model has been evaluated by several measurement techniques and gave very well accuracy, sensitivity, specificity, efficiency and ROC-AUC value results.

Keywords: *CNN, SERS spectra, SARS-CoV-2 proteins, machine learning, classification.*

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Application of sphere packing to parametric linear programming

Lutbat Yadamsuren[1,*], Bayartugs Tamjav[1], Ulziibayar Vandandoo[1], Enkhbat Rentsen[2]

[1]*Department of Mathematics, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[2]*Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia*

**Correspondence author. Email: lutaa@must.edu.mn, ORCID: 0000-0002-8956-9134*

ABSTRACT
In this talk, we consider the application of sphere packing, which is to pack non-overlapping spheres with the maximum volume into convex set, to the parametric linear programming. We propose a finite algorithm which finds a range of parameters for solving the problem based on general sphere packing problem. The proposed algorithm was tested on several parametric linear programming examples.

Keywords: *Sphere packing, linear programming, Convex set.*

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Dynamic aspects of the SEIR epidemic model incorporating psychological influence

Ghaus ur Rahman[1,*], Saad Ihsan Butt[2], Dumitru Baleanu[3], Rahim Uddin[1]

[1]*Department of Mathematics and Statistics, University of Swat, Pakistan*
[2] *COMSATS University Islamabad, Lahore Campus, Pakistan*
[3]*Department of Mathematics, Cankaya University, Ankara, Turkey*

**Correspondence author. Email: dr.ghaus@uswat.edu.pk, ORCID: 0000-0002-0988-2455*

ABSTRACT

The present article explores the dynamics of a SEIR type epidemic model while incorporating a non-monotonic incidence rate. Physically, it depicts the psychological effects of various severe diseases on people when the number of infected people rapidly increases. Moreover, a new class of exposed population is introduced in SIR existing model, and the psychological impact of diseases is studied in this research endeavour. To do this, a particular uptake function is used to capture this scenario. Studying the dynamical aspects of the newly formulated model, it will be shown that as time evolves, either the community of infected individuals will approach zero or the disease will persist. After expounding the feasible equilibrium points and basic reproductive number, it will be shown that the model under consideration is attracting. The Dulac function is formulated to show that our proposed model does not show nontrivial periodic orbits. Certain conditions will be developed to show that the trivial equilibrium point is the hyperbolic saddle, saddle node, and stable hyperbolic node, respectively. Afterward, it will be shown that under some circumstances, the model fulfils the criteria of local and global asymptotic stability at the feasible equilibrium points.

Keywords: *Dulac function, Stability, Epidemic model, Non-monotone incidence rate, Psychological impact.*

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Using Hyers-Ulam stability for Tarig Transform of linear differential equations and its applications

Dr. N. Subashini Narayanaswamy[1,*]

[1]*Department of mathematics Sathyabama institute of science and technology, Semmancherry, Chennai, Tamilnadu, India*

**Correspondence author. Email: subashini.sams@gmail.com*

ABSTRACT

In this manuscript, we present the motivation of Tarig transform for homogeneous and non-homogeneous linear differential equation. With the help of this new integral transform we solve higher order Tarig transform of linear differential equation. It outlined, in other words, the prerequisites for Hyers-Ulam stability in the Tarig transform. This is the first attempt to demonstrate the stability of a linear differential equation using the Tarig transform. The results of this study also demonstrated that the Tarig transform method is more practical for examining the stability issue for differential equations with constant coefficients. Applications are then discussed to demonstrate our methodology.

Keywords: *Differential equation (DE), Hyers-Ulam stability (HUS), Tarig transforms (TT).*

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Some new subclasses of bi-univalent functions defined by convolution associated with linear differential operator

Prema S[1,*], Bhuvaneswari Raja[2], N. Subashini Narayanaswamy[3]

[1]*Department of Mathematics, Rajalakshmi Institute of Technology, Chennai, Tamil Nadu, India*
[2]*Department of Mathematics, Aalim Muhammed Salegh College of Engineering, Chennai, India*
[3]*Department of Mathematics, Sathyabama Institute of Science and Technology, Semmancherry, Chennai, Tamilnadu, India*

**Correspondence author. Email: premnehaa@gmail.com, ORCID: 0000-0001-9967-5988*

ABSTRACT

The main object of this paper is investigating a new subclass of bi-univalent function in the open unit disk U which is defined by convolution of Al-Oboudi Differential Operator. And obtained the initial two Taylor-McLaurin co-efficient $|a_2|$ and $|a_3|$ for the subclass $S_{\Sigma,r}^{m,n,b,\delta}$ of Bi-Univalent function.

Keywords: *Analytic functions, univalent functions, bivalent functions, convolution and Al-Oboudi differential operator.*

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Main difficulties in computing the Schwartz–Christoffel integral

Soninbayar Jambaa[1,*]

[1] *Department of Applied Mathematics, School of Engineering and Applied Sciences, National University of Mongolia, Ulaanbaatar, Mongolia*

**Correspondence author. Email: soninbayar@seas.num.edu.mn*

ABSTRACT

In this work, we consider the Christoffel-Schwartz integral, a numerical method for determining the inverse images of the vertices of rectilinear polygons and its application to practical conformal mapping, numerical integration, in the MatLab program system, the method of P.P. Kufarev, which translates to problems of dynamical systems.

Keywords: *Schwartz-Christoffel formulas, method P.P. Kufareva, numerical method, dynamical systems.*

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Multiplicative optimal control problem

Enkhbat Rentsen[1], Bayartugs Tamjav[2,*], Ulziibayar Vandandoo[3]

[1]*The Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia*
[2]*Department of Mathematics, School of Science and Technology, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*
[3]*School of Applied Sciences, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*

**Correspondence author. Email: bayartugs@must.edu.mn, ORCID: 0000-0002-8956-9134*

ABSTRACT
In this paper, we consider a multiplicative optimal control problem subject to a system of linear differential equation. It has been shown that product of two concave functions defined positively over a feasible set is quasiconcave. It allows us to consider the original problem from a view point of quasiconvex maximization theory and algorithm. Global optimality conditions use level set of the objective function and convex programming as subproblem. The objective function is product of two concave functions. We consider minimization of the objective functional. The problem is nonconvex optimal control and application of Pontriyagin's principle does not always guarantee finding a global optimal control. Based on global optimality conditions, we develop an algorithm for solving the minimization problem globally.

Keywords: *Quasiconvex, Pontriyagin's principle, convex programming, optimal control.*

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Time effect of light signals on traffic speed

Ganzorig Dashdorj[1,*]

[1]*Department of Mathematics, School of Applied Sciences, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*

**Correspondence author. Email: daganzorig@must.edu.mn, ORCID: 0009-0004-3702-0870*

ABSTRACT

More than 80% of all vehicles registered in Mongolia are located in Ulaanbaatar, the capital of the country, and 150-200 vehicles are added to the traffic every day. Although there is a center for monitoring and regulating the movement of vehicles with the help of cameras, it is still only responsible for controlling the traffic with traffic lights in case of traffic jams and traffic jams, and not for any investigation. The purpose of this work is to study how much traffic intensity changes by changing the time of traffic lights.

Keywords: *Waves, propagation, distribution, average speed.*

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Future development trend of Sukhbaatar district, Ulaanbaatar city

Altankhuyag Buyankhishig[1,*], Agvaanjigmid Bolormaa[2]

[1]*Department of Public Administration, Office of the Governor of Sukhbaatar District, Ulaanbaatar, Mongolia*
[2]*Departmen of Social Sciences, School of Business Administration and Humanities, Mongolian University of Science and Technology, Ulaanbaatar, Mongolia*

**Correspondence author. Email: buyankhishig1213@gmail.com*

ABSTRACT

In this study, the main social, economic and environmental indicators of Sukhbaatar district of Ulaanbaatar city, their perspective until 2030, development, population growth, migration, poverty, employment, etc. and how planning in turn affects society, economy, and the environment is modeled. Sukhbaatar district is located in the central part of Ulaanbaatar city with many buildings and densely populated areas that have an important influence on the development of the city Sukhbaatar district is an attractive and convenient district with greenery, ancient heritage streets, cultural service centers, trade, tourism, and opportunities. It is possible to develop the tourism and trade services, small and medium industry development sectors as the priority areas of economic development of the district. Based on about 70 indicators of Sukhbaatar district's population and schools, the dynamic model of the district's strategy and development was developed on the "Vensim" program, and the various policy options for population localization, appropriate level of settlement development, appropriate employment, and welfare services were compared and planned. will be possible. In the dynamic population model, the population's future growth and prospects were modeled using the numerical data of births, deaths, internal and external migration of the region from 2014 to 2022.

Keywords: *Tourism, green buildings, socio-economy, modelling.*

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On one optimization method for linearly controlled systems with constraints

Dmitry Trunin [1,*]

[1]*Buryat State University, Ulan-Ude, Russia*

**Correspondence author. Email: tdobsu@yandex.ru, ORCID: 0000-0003-2431-973X*

ABSTRACT
In linearly controlled systems with constraints, new conditions for nonlocal improvement of controls are constructed in the form of a fixed point problem. To solve the constructed fixed point problem, a special iterative algorithm is proposed, based on the operation of projecting onto a set of control values. On the basis of the proposed control improvement algorithm, a method for solution of the optimal control problem is constructed.

Keywords: *nonlinear controlled system, terminal constraints, control improvement conditions, fixed point problem, iterative algorithm.*

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Positivity and elementary stability: extended NSFD scheme for the SEIR epidemic model

Enkh-Amar Shagdar[1,*], Batgerel Balt[1]

[1]*Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia*

**Correspondence author. Email: enkhamar_sh@mas.ac.mn, ORCID: 0000-0001-5305-2021*

ABSTRACT

When constructing a non-standard difference scheme for the differential equations, denominator of the discrete derivative is chosen as the functions depending on step-sizes on the computational grid or lattice. In other existing non-standard finite difference methods for SEIR epidemic model, those denominator functions have the same. The new scheme discussed in this article is characterized by the fact that the corresponding derivatives of the system of ordinary differential equations are replaced by different denominator functions depending on each equation. The proposed method has important properties such as conversation law, positivity and it preserves elementary stability. By numerical comparisons are confirmed that the accuracy of new method is better than that of standard and non-standard finite difference schemes (Mickens-type NSFD schemes with the same denominator functions). In addition, we have developed a positivity and elementary stability analysis for the new scheme.

Keywords: *Nonstandard Finite Difference Scheme (NSFD), positivity, elementary stability.*

Maximum principle methods based on fixed point problems

Alexander Buldaev [1,*], Ivan Kazmin[1]

[1] *Buryat State University, Ulan-Ude, Russia*

**Correspondence author. Email: buldaev@mail.ru, ORCID: 0000-0003-2052-9430*

ABSTRACT

New forms of optimality conditions in the form of fixed-point problems, equivalent to the known conditions of the maximum principle, are proposed for the class of linear optimal control problems. New forms of conditions of the maximum principle allow to construct new methods of search of the controls satisfying the maximum principle. The proposed fixed point methods do not contain the time-consuming of parametric control at each iteration, which is typical for gradient methods. In the proposed methods, two Cauchy problems are solved at each iteration, and it is possible to improve control.

Keywords: *optimal control problem, maximum principle, fixed point problem, iterative method.*

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Polynomial WENO reconstruction

Ariunaa Uuriintsetseg [1,*], Michael Dumbser[2]

[1]*Department of Mathematics, School of Applied Sciences, Mongolian university of Science and Technology, Ulaanbaatar, Mongolia*
[2]*Laboratory of Applied Mathematics, Department of Civil, Environmental and Mechanical Engineering University of Trento, Trento, Italy*

**Correspondence author. Email: ariunaa@must.edu.mn, ORCID: 0000-0001-8754-872X*

ABSTRACT
The original pointwise WENO reconstruction of Jiang and Shu described for 1D is rather difficult to generalize to unstructured triangular and tetrahedral meshes in two and three space dimensions because of the need to determine the optimal linear weights. Therefore, we present a different idea which can be extended very easily to the unstructured case. The alternative reconstruction procedure described here for the one-dimensional case follows directly from the general guidelines for unstructured triangular and tetrahedral meshes in two and three space dimensions. The polynomial WENO reconstruction operator entire polynomials, as the ENO approach proposed by Harten et al. However, we formally write our method like a WENO scheme with a particularly simple choice for the linear weights. The most important difference of our approach compared to the classical WENO schemes of Jiang and Shu is that standard WENO methods reconstruct point values at the Gaussian integration points instead of an entire polynomial valid inside each control volume. The advantage of the polynomial WENO reconstruction is its straight–forward extension to general unstructured meshes. The drawback is that at a given order of accuracy k the total stencil needed for the reconstruction is bigger than the one of the classical pointwise WENO scheme.

Keywords: *Unstructured mesh, WENO.*

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Methods for finding extremal controls in discrete-continuous control systems

Alexander Buldaev[1,*], Vladislav Dumnov[1]

[1]*Buryat State University, Ulan-Ude, Russia*

**Correspondence author. Email: buldaev@mail.ru, ORCID: 0000-0003-2052-9430*

ABSTRACT

A class of discrete-continuous control systems is considered, which are described by differential equations with piecewise constant controls. Control optimality conditions are constructed in the form of fixed point problems in the control space. Iterative methods are constructed to search for controls that satisfy the obtained optimality conditions. The proposed methods do not require the calculation of the values of the control optimality criterion at each iteration, in contrast to the methods of the gradient type. The methods are characterized by the absence of a search procedure for an improving control at each iteration and by non-local successive approximations of the control.

Keywords: *Discrete-continuous system, piecewise constant control, optimality conditions, fixed point problem, iterative method.*

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A two-stage approach for beam hardening artifact reduction in low-dose dental CBCT

Taigyntuya Bayaraa[1,*]

[1]*Department of Mathematics, School of Applied Sciences, Mongolian university of Science and Technology, Ulaanbaatar, Mongolia*

**Correspondence author. Email: taigyntuya@must.edu.mn*

ABSTRACT

We proposed a two-stage method for beam hardening artifact correction of dental cone beam computerized tomography (CBCT). Compared to standard CT, the additional difficulty of dental CBCT comes from the problems caused by offset detector, FOV truncation, and low signal-to-noise ratio due to low X-ray irradiation. To address these problems, the proposed method primarily performs a sinogram adjustment in the direction of enhancing data consistency, considering the situation according to the FOV truncation and offset detector. This sinogram correction algorithm significantly reduces beam hardening artifacts caused by high density materials such as teeth, bones, and metal implants, while tending to amplify special types of noise. To suppress such noise, a deep convolutional neural network is complementarily used, where CT images adjusted by the sinogram correction are used as the input of the neural network. Numerous experiments validate that the proposed method successfully reduces beam hardening artifacts and, in particular, has the advantage of improving the image quality of teeth, associated with maxillofacial CBCT imaging.

Keywords: *Computerized tomography, CT, FOV, low X-ray irradiation*

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